

Assisted chaotic inflation in brane-world cosmology

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Assisted chaotic inflation in brane cosmology is discussed. We work in the framework of Randall-Sundrum (RS) II model, in which adopting the RS condition the only parameter is the five-dimensional Planck mass. Using the scalar spectral index and the amplitude of scalar perturbations we determine both the mass of the scalar fields responsible for inflation and the fundamental Planck mass of the higher-dimensional theory. We find that the mass of the scalars has the typical value of the inflaton mass in chaotic inflation ($M_I \sim 10^{13}$ GeV) and that the five-dimensional Planck mass is very close to the GUT (Grand Unified Theories) scale ($M_5 \sim (10^{16} - 10^{17})$ GeV). Furthermore, no matter how many scalar fields we use it is not possible to have chaotic inflation with field values below the fundamental Planck mass.

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Inflation [1] has become the standard paradigm for the early Universe, because it solves some outstanding problems present in the standard Hot Big-Bang cosmology, like the flatness and horizon problems, the problem of unwanted relics, such as magnetic monopoles, and produces the cosmological fluctuations for the formation of the structure that we observe today. The recent spectacular CMB data from the WMAP satellite [2] have strengthened the inflationary idea, since the observations indicate an *almost* scale-free spectrum of Gaussian adiabatic density fluctuations, just as predicted by simple models of inflation. According to chaotic inflation with a potential for the inflaton field ϕ of the form $V = (1/2)m^2\phi^2$, the WMAP normalization condition requires for the inflaton mass m that $m = 1.8 \times 10^{13}$ GeV [3]. However, a yet unsolved problem about inflation is that we do not know how to integrate it with ideas in particle physics. For example, we would like to identify the inflaton, the scalar field that drives inflation, with one of the known fields of particle physics. Furthermore, it is important that the inflaton potential emerges naturally from an underline fundamental theory. See however [4] as an example of realistic embedding withing particle physics. In this model the inflaton has Standard Model gauge charges instead of being a ad-hoc gauge singlet added by hand.

It is known [5] that if there is just a single scalar field ϕ , the exponential potential $V(\phi) = V_0 \exp(-\lambda\phi/m_{pl})$, where m_{pl} is Planck mass and $\lambda = \sqrt{16\pi/p}$, leads to power-law solution $a(t) \sim t^p$, with $a(t)$ the scale factor of the universe as a function of the cosmic time. For steep potentials ($p < 1$) the solution is decelerating and therefore it does not correspond to inflation, while for shallow potentials ($p > 1$) the solution is accelerating and thus it is an inflationary solution. Scalar potentials of exponential form emerge in supergravity theories, however these theories typically predict steep exponential potentials. In assisted inflation [6] many scalar

fields drive inflation in a cooperative way and so it is possible to obtain inflationary solutions even in such models.

Over the last years the brane-world models have been attracting a lot of attention as a novel higher-dimensional theory. Brane models are inspired from M/string theory and although they are not yet derivable from the fundamental theory, at least they contain the basic ingredients, like extra dimensions, higher-dimensional objects (branes), higher-curvature corrections to gravity (Gauss-Bonnet) etc. Since string theory claims to give us a fundamental description of nature it is important to study what kind of cosmology it predicts. Furthermore, despite the fact that inflationary models have been analyzed in standard four-dimensional cosmology, it is challenging to discuss them in alternative gravitational theories as well. In brane-world models it is assumed that the standard model particles are confined on a 3-brane while gravity resides in the whole higher dimensional spacetime. The model first proposed by Randall and Sundrum (RSII) [7], is a simple and interesting one, and its cosmological evolutions have been intensively investigated. An incomplete list can be seen e.g. in [8]. In the present work we would like to study assisted chaotic inflation in the framework of RSII model. According to that model, our 4-dimensional universe is realized on the 3-brane with a positive tension located at the UV boundary of 5-dimensional AdS spacetime. In the bulk there is just a cosmological constant Λ_5 , whereas on the brane there is matter with energy-momentum tensor $\tau_{\mu\nu}$. Also, the five dimensional Planck mass is denoted by M_5 and the brane tension is denoted by T .

We start by presenting the theoretical framework in which we will be working, namely the RSII brane model and the dynamics of assisted inflation, as well as the two quantities (spectral index and amplitude of perturbations) that will link the theoretical model to observations.

If Einstein's equations hold in the five dimensional bulk, then it has been shown in [9] that the effective four-dimensional Einstein's equations induced on the brane

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can be written as

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \frac{8\pi}{m_{pl}^2} \tau_{\mu\nu} + \left(\frac{1}{M_5^3}\right)^2 \pi_{\mu\nu} - E_{\mu\nu} \quad (1)$$

where $g_{\mu\nu}$ is the induced metric on the brane, $\pi_{\mu\nu} = \frac{1}{12} \tau \tau_{\mu\nu} + \frac{1}{8} g_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{4} \tau_{\mu\alpha} \tau_\nu^\alpha - \frac{1}{24} \tau^2 g_{\mu\nu}$, Λ_4 is the effective four-dimensional cosmological constant, m_{pl} is the usual four-dimensional Planck mass and $E_{\mu\nu} \equiv C_{\beta\rho\sigma}^\alpha n_\alpha n^\rho g_\mu^\beta g_\nu^\sigma$ is a projection of the five-dimensional Weyl tensor $C_{\alpha\beta\rho\sigma}$, where n^α is the unit vector normal to the brane. The tensors $\pi_{\mu\nu}$ and $E_{\mu\nu}$ describe the influence of the bulk in brane dynamics. The five-dimensional quantities are related to the corresponding four-dimensional ones through the relations

$$m_{pl} = 4 \sqrt{\frac{3\pi}{T}} M_5^3 \quad (2)$$

and

$$\Lambda_4 = \frac{1}{2M_5^3} \left(\Lambda_5 + \frac{T^2}{6M_5^3} \right) \quad (3)$$

In a cosmological model in which the induced metric on the brane $g_{\mu\nu}$ has the form of a spatially flat Friedmann-Robertson-Walker model, with scale factor $a(t)$, the Friedmann-like equation on the brane has the generalized form [8]

$$H^2 = \frac{\Lambda_4}{3} + \frac{8\pi}{3m_{pl}^2} \rho + \frac{1}{36M_5^6} \rho^2 + \frac{C}{a^4} \quad (4)$$

where C is an integration constant arising from $E_{\mu\nu}$. The cosmological constant term and the term linear in ρ are familiar from the four-dimensional conventional cosmology. The extra terms, i.e the “dark radiation” term and the term quadratic in ρ , are there because of the presence of the extra dimension. Adopting the Randall-Sundrum fine-tuning

$$\Lambda_5 = -\frac{T^2}{6M_5^3} \quad (5)$$

the four-dimensional cosmological constant vanishes. In addition, the dark radiation term will be quickly diluted during inflation and therefore we shall neglect it. So the generalized Friedmann equation takes the final form

$$H^2 = \frac{8\pi G}{3} \rho \left(1 + \frac{\rho}{\rho_0} \right) \quad (6)$$

where

$$\rho_0 = 96\pi G M_5^6 \quad (7)$$

with G the Newton’s constant. One can see that the evolution of the early universe can be divided into two eras. In the low-energy regime $\rho \ll \rho_0$ the first term dominates and we recover the usual Friedmann equation of the conventional four-dimensional cosmology. In the

high-energy regime $\rho_0 \ll \rho$ the second term dominates and we get an unconventional expansion law for the universe. Since inflation is assumed to take place in the high-energy regime we shall keep only the term which is quadratic in ρ and from now on we shall make use of the new law for expansion of the universe

$$H(\rho) = \frac{\rho}{6M_5^3} \quad (8)$$

Assisted inflation model [6] involves more than one scalar fields ϕ_i ($i = 1, 2, \dots, p$). Let us present here the dynamics of this model for chaotic inflation, i.e. a simple quadratic potential $V(\phi) = m^2 \phi^2/2$. We consider the case in which all scalars have the same potential (i.e. same mass). The scalar fields are taken to be noninteracting but are minimally coupled to gravity. Under these assumptions the total scalar potential is $W(\phi_1, \dots, \phi_p) = \sum_{i=1}^p V(\phi_i)$ and the equations of motion are

$$\ddot{\phi}_i = -m^2 \phi_i - 3H\dot{\phi}_i \quad (9)$$

$$H = \frac{1}{6M_5^3} \sum_{i=1}^p \left(V(\phi_i) + \frac{\dot{\phi}_i^2}{2} \right) \quad (10)$$

To simplify things we shall consider a particular solution in which all fields are taken equal $\phi_1 = \phi_2 = \dots = \phi_p = \Phi$. The simplified equations in the slow-roll approximation become

$$0 = -m^2 \Phi - 3H\dot{\Phi} \quad (11)$$

$$H = \frac{1}{6M_5^3} p V(\Phi) \quad (12)$$

The amplitude of scalar perturbations is defined by

$$A_s = \frac{2}{5} \mathcal{P}_{\mathcal{R}}^{1/2} \quad (13)$$

where $\mathcal{P}_{\mathcal{R}}$ is the spectrum of the curvature perturbation given by [10]

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi} \right)^2 \delta_{ij} \frac{\partial N}{\partial \phi_i} \frac{\partial N}{\partial \phi_j} \quad (14)$$

with N the number of e-folds remaining (defined by $N = -\int dt H$), which satisfies the useful equation

$$\sum_{i=1}^p \frac{\partial N}{\partial \phi_i} \dot{\phi}_i = -H \quad (15)$$

Furthermore, the spectral index is given by [10]

$$n - 1 = 2 \frac{\dot{H}}{H^2} - 2 \frac{\frac{\partial N}{\partial \phi_i} \frac{\partial N}{\partial \phi_j} \left(\frac{8\pi \dot{\phi}_i \dot{\phi}_j}{m_{pl}^2 H^2} - \frac{W_{i,j}}{3H^2} \right)}{\delta_{ij} \frac{\partial N}{\partial \phi_i} \frac{\partial N}{\partial \phi_j}} \quad (16)$$

In our case in which $H(\rho) = \rho/(6M_5^3)$, $V(\phi) = m^2\phi^2/2$ and all fields are taken to be equal we obtain

$$\delta_{ij} \frac{\partial N}{\partial \phi_i} \frac{\partial N}{\partial \phi_j} = \frac{H^2}{p \dot{\Phi}^2} \quad (17)$$

$$\frac{\partial N}{\partial \phi_i} \frac{\partial N}{\partial \phi_j} \dot{\phi}_i \dot{\phi}_j = H^2 \quad (18)$$

$$W_{i,j} = m^2 \delta_{i,j} \quad (19)$$

$$N(\Phi) = \frac{p m^2 \Phi^4}{192 M_5^6} \quad (20)$$

We fix the number of e-folds to be $N_* = 60$ and allow for the number p of the scalar fields to take values $p = 2, 10, 50, 100$. In our model there are just two free parameters, namely the five-dimensional Planck mass M_5 and the mass m of the scalar fields. We take for $A_s = 2 \times 10^{-5}$ and $n = 0.95$. From these two observational facts we can determine both M_5 and m . First, from the amplitude A_s we determine the ratio m/M_5 and then from the spectral index we compute M_5 . We use the formulae

$$\frac{m}{M_5} = \left(\frac{15\pi A_s}{p^{5/4} (192 N_*)^{1/4}} \right)^{2/3} \quad (21)$$

$$n = 1 - \frac{1}{2p N_*} - \sqrt{\frac{p}{3N_*}} \frac{24\pi}{p^2} \left(\frac{M_5}{m_{pl}} \right)^2 \frac{M_5}{m} \quad (22)$$

Our results can be shown below

- For $p = 2$, $M_5 = 6.24 \times 10^{16}$ GeV, $m = 7.08 \times 10^{13}$ GeV
- For $p = 10$, $M_5 = 1.11 \times 10^{17}$ GeV, $m = 3.30 \times 10^{13}$ GeV
- For $p = 50$, $M_5 = 1.90 \times 10^{17}$ GeV, $m = 1.48 \times 10^{13}$ GeV
- For $p = 100$, $M_5 = 2.40 \times 10^{17}$ GeV, $m = 1.05 \times 10^{13}$ GeV

One can see that the mass m is of the order $\sim 10^{13}$ GeV, which is the typical inflaton mass in chaotic inflation with just one scalar field. In addition, we see that the fundamental Planck mass is close to the GUT

scale, $M_{GUT} \sim 10^{16}$ GeV. Assisted inflation typically involves a large number of scalar fields, for which we obtain $M_5 \sim 10^{17}$ GeV.

In the case of standard four-dimensional cosmology, assisted inflation is used in order to have chaotic inflation with sub-Planckian field values. The main motivation for assisted inflation in a brane-world scenario would be to have inflation with field values below the fundamental Planck mass. We now compute the minimum number of scalar fields for which $\Phi_* < M_5$ (if possible). We make use of the formula (another way of writing equation (20) for $N_* = 60$)

$$\frac{\Phi_*}{M_5} = \left(\frac{60 \times 192}{p} \right)^{1/4} \left(\frac{M_5}{m} \right)^{1/2} \quad (23)$$

Then using equation (21) we obtain that Φ_*/M_5 goes like $p^{1/6}$ and it is always larger than unity. Therefore it is impossible to have chaotic inflation with field values below M_5 .

To summarize, in the present work we have discussed assisted chaotic inflation in the RSII brane-world scenario. In this model the universe expands according to a novel Friedmann-like equation and inflation is driven by p non-interacting scalar fields which are minimally coupled to gravity. For simplicity we have considered the case in which all fields are taken to be equal with a common quadratic potential $V(\phi) = m^2\phi^2/2$. Using the observational facts that $A_s = 2 \times 10^{-5}$, $n = 0.95$ we have determined the two free parameters of the model, i.e. the mass m of the scalar fields and the five-dimensional Planck mass M_5 . Our results show that $m \sim 10^{13}$ GeV, a typical inflaton mass in chaotic inflation with just a single scalar field, and that $M_5 \sim (10^{16} - 10^{17})$ GeV. Furthermore, we have found that it not possible to have chaotic inflation with field values below the fundamental Planck mass.

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